

# A

First Round: February 25, 2017 at Regional Testing Centers  
Second Round: April 8, 2017 at The University of North Alabama

## GEOMETRY EXAMINATION

Construction of this test directed  
by  
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### INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate

## Why Major in Mathematics?

**What sorts of jobs can I get with a mathematics degree?** Examples of occupational opportunities available to math majors:

Market Research Analyst	Cryptanalyst	Mathematician
Air Traffic Controller	Professor	Meteorologist
Climate Analyst	Pollster	Medical Doctor
Estimator	Population Ecologist	Lawyer
Research Scientist	Operations Research	Actuary
Computer Programmer	Data Mining	Statistician

**Where can I work?** What sorts of companies hire mathematicians? Well just to name a few...

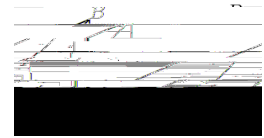
**U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.

**Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.

**Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and



13. In the right triangle  $\triangle ABC$  shown,  $D$  is on  $\overline{AC}$ ,  $m\angle BAD = 30^\circ$ ,  $m\angle BDC = 60^\circ$  and  $AD = 4$ . Find  $BC$ .
- (A) 2 (B)  $2\sqrt{2}$  (C)  $3\sqrt{2}$  (D)  $2\sqrt{3}$  (E) None of these



14. The point  $(2;1)$  is reflected about the line  $y = 2x$ . What are the coordinates of the resulting point?
- (A)  $(-0.5;2)$  (B)  $(-0.3;2.1)$  (C)  $(-0.2;2.8)$  (D)  $(-0.4;2.2)$  (E) None of these

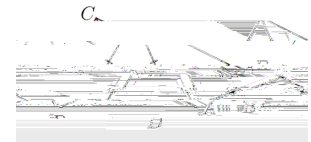
15. Which of the following statements is false?

- (A) A parallelogram is a trapezoid. (B) A square is a parallelogram.  
 (C) A kite is a quadrilateral. (D) A rectangle is a parallelogram. (E) Some rhombi are rectangles.

16. A circle and a parabola are drawn on a piece of paper. What is the maximum number of regions they divide the paper into?

- (A) 2 (B) 4 (C) 6 (D) 8 (E) None of these=11

17. Triangle  $\triangle ABC$  is shown divided into three isosceles triangles,  $\triangle BCD$ ,  $\triangle CDE$  and  $\triangle DEA$ . Given that  $BC = CD$ ,  $CD = DE$ ,  $DE = EA$  and  $m\angle A = 20^\circ$ , find  $m\angle B$ .



- (A) 20 (B) 40 (C) 60 (D) 80 (E) None of these

18. A segment has one endpoint of  $(-3;4)$  and a midpoint of  $(2;3)$ . What are the coordinates of other endpoint?

- (A)  $(7;2)$  (B)  $(\frac{1}{2}; \frac{7}{2})$  (C)  $(4; -3)$  (D)  $(2; \frac{11}{2})$  (E) None of these

19. In the following diagram, parallel lines  $n$  and  $m$  are cut by two perpendicular

- (A) 20 (B) 20

23. Two circles of radii 2 and 3 are externally tangent. A third circle is circumscribed about the two smaller circles. A pin is dropped inside the largest circle. What

33. One rainy afternoon, you decide you want to start a craft project: make the Deathly Hallows (pictured) out of metal wire. The Deathly Hallows consists of a circle inscribed in an equilateral triangle with a single altitude. If you want a side of the triangle to be 2 inches, how much wire will it take to make the Deathly Hallows, in inches?



(A)  $\frac{18 + 3\sqrt{3}}{3}$

(B)  $\frac{18 + 3\sqrt{3} + 2\sqrt{3}}{3}$

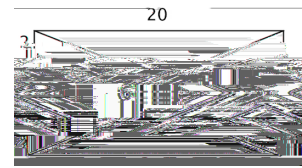
(C)  $\frac{21 +}{3}$

(D)  $\frac{21 + 2\sqrt{3}}{3}$

(E) None of these

34. A line containing points  $(2;1)$  and  $(8;m)$  is parallel to the line containing points  $(7;m + 1)$  and  $(11;1)$ . Find the value of  $m$

41. Amelia's front yard, which measures 20 feet wide by 10 feet long, is to have two diagonal concrete sidewalks put in, as shown below. If the sidewalks measure 2 feet along the side of the yard, how much of Amelia's yard will not be concrete?



- (A)  $50 \text{ ft}^2$  (B)  $125 \text{ ft}^2$  (C)  $200 - 8\sqrt{29} \text{ ft}^2$   
 (D)  $200 - 20\sqrt{5} \text{ ft}^2$  (E) None of these
42. A regular square pyramid has height  $\sqrt{15}$  with the area of a triangular face equal to 10. Find the surface area of the pyramid.
- (A)  $40 + \sqrt{15}$  (B)  $40 + 2\sqrt{15}$  (C)  $60$  (D) 120 (E) None of these
43. In  $\triangle ABC$  shown,  $D$  is the midpoint of  $\overline{AB}$  and  $E$  is the midpoint of  $\overline{AC}$ . If  $BC = 7x + 1$  and  $DE = 4x - 2$ , then find the length  $BC$ .
- (A) 10 (B) 18 (C) 22 (D)  $36$  (E) None of these
44. If 5 and 9 are two sides of a right triangle, which of the following could be the third side?
- (A)  $\sqrt{24}$  (B) 5 (C) 7 (D)  $\sqrt{56}$  (E) None of these
45. In  $\triangle ABC$  shown,  $\angle ACB$  and  $\angle BED$  are right angles,  $m\angle EAD = 25^\circ$  and  $m\angle BDC = 40^\circ$ . If  $BC = 4$ , find  $ED$ .
- (A)  $\frac{4 \sin 15^\circ}{\sin 40^\circ}$  (B)  $\frac{4 \sin 75^\circ}{\sin 50^\circ}$  (C)  $\frac{\sin 40^\circ}{4 \sin 15^\circ}$  (D)  $\frac{\sin 50^\circ}{4 \sin 75^\circ}$  (E) 2
46. In  $\triangle ABC$  with right angle at  $C$ , altitude  $\overline{CH}$  and median  $\overline{CM}$  trisect the right angle. If the area of  $\triangle CHM$  is  $K$ , then the area of  $\triangle ABC$  is:
- (A)  $6K$  (B)  $4\sqrt{3}K$  (C)  $3\sqrt{3}K$  (D)  $3K$  (E)  $4K$
47. Points  $A, B, C, D$  and  $E$  are drawn on a circle and connected to form a five-pointed star. Find the value of  $m\angle A + m\angle B + m\angle C + m\angle D + m\angle E$ .
- (A) 120 (B)  $180$  (C) 240 (D) 360 (E) None of these
48. In  $\triangle ABC$  with right angle at  $C$ , midpoints  $D$  and  $E$  lie along  $\overline{AC}$  and  $\overline{BC}$ , respectively. If  $AC = 6$  and  $BC = 8$ , find the height of trapezoid  $ABED$ .
- (A)  $2.4$  (B) 3.5 (C)  $\sqrt{5}$  (D)  $\sqrt{7}$  (E) None of these
49.  $ABCD$  is a square with side of length 1. Points  $E$  and  $F$  are taken respectively on sides  $\overline{AB}$  and  $\overline{AD}$  so that  $AE = AF$  and the quadrilateral  $CDFE$  has maximum area. What is the maximum area of  $CDFE$ ?
- (A)  $\frac{5}{8}$  (B)  $\frac{23}{32}$  (C)  $\frac{7}{8}$  (D)  $\frac{9}{10}$  (E) None of these
50. The angles of a convex, irregular pentagon form an arithmetic sequence. If all angles have integer measure, what is the largest measure that the smallest angle can have in degrees?
- (A) 100 (B) 106 (C) 107 (D) 110 (E)  $None\ of\ these=108$