

# A

First Round: February 24, 2018 at Regional Testing Centers  
Second Round: April 14, 2018 at The University of Alabama at Birmingham

## COMPREHENSIVE EXAM

Construction of this test directed  
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### INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate

## Why Major in Mathematics?

**What sorts of jobs can I get with a mathematics degree?** Examples of occupational opportunities available to math majors:

Market Research Analyst	Cryptanalyst	Mathematician
Air Traffic Controller	Professor	Meteorologist
Climate Analyst	Pollster	Medical Doctor
Estimator	Population Ecologist	Lawyer
Research Scientist	Operations Research	Actuary
Computer Programmer	Data Mining	Statistician

**Where can I work?** What sorts of companies hire mathematicians? Well just to name a few...

**U.S. Government Agencies** such as the National Center for Computing Sciences, the National Institute of Standards and Technology (NIST), the National Security Agency (NSA), and the U.S. Department of Energy.

**Government labs and research offices** such as Air Force Office of Scientific Research, Los Alamos National Laboratory, and Sandia National Laboratory.

**Engineering research organizations** such as AT&T Laboratories - Research, Exxon Research and Engineering, and IBM Research.

**Computer information and software firms** such as Adobe, Google, Mentor Graphics, Microsoft, and Yahoo Research.

**Electronics and computer manufacturers** such as Alcatel-Lucent, Hewlett-Packard, Honeywell, Philips Research, and SGI.

**Aerospace and transportation equipment manufacturers** such as Boeing, Ford, General Motors, and Lockheed Martin.

**Transportation service providers** such as FedEx Corporation and United Parcel Service (UPS).

**Financial service and investment management firms** such as Citibank, Morgan Stanley, and Prudential.

### **A Mathematics Major isn't just for those wanting to be Mathematicians!**

The top scoring major on the Law School Entrance Exam (LSAT) is Mathematics (Source: Journal of Economic Education)

Mathematics is also a top 5 scoring major on the Medical School Entrance Exam (MCAT) (Source: American Institute of Physics)

Study in the field of mathematics offers an education with an emphasis on careful problem solving, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, and health and environmental fields require mathematical techniques for their solutions. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems. The University of North Alabama offers an undergraduate degree in Mathematics and has many great things to offer, including a new

1. The measures of the exterior angles of a hexagon are  $x$ ,  $2x$ ,  $3x$ ,  $3x$ ,  $4x$ , and  $5x$ . Find the measure of the largest interior angle.

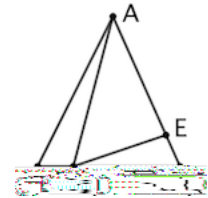
- (A) 20 (B) 100 (C) 120 (D) 160 (E) None of these

2. There is exactly one integer  $a$  for which the polynomial  $f(x) = ax^4 + 15x^3 - 5x^2 + 10x - a$  is divisible by  $x + 3$ . Find the sum of the value  $a$  and the coefficient on the  $x$  term of the quotient.

- (A) 10 (B) 16 (C) 20 (D) 24 (E) None of these

3. In the triangle shown, the measure of  $\angle BAD = 38^\circ$ ,  $AB = AC$ , and  $AD = AE$ . Find the measure of  $\angle CDE$ .

- (A) 19 (B) 27 (C) 38 (D) 52 (E) None of these



4. Find the sum of the solutions to the equation  $16^x - 7(4^x) = 10$ .

- (A)  $\log_4 7$  (B)  $\log_4 10$  (C)  $\log_{16} 7$  (D)  $\log_{16} 10$  (E) None of these

5. A number is *abundant* if the sum of its proper divisors is greater than the number itself. Recall that a proper divisor of a number  $n$  is any positive divisor which is less than  $n$ . Which of the following is an abundant number?

- (A) 16 (B) 20 (C) 22 (D) 28 (E) 32

6. Find the number of solutions  $(a; b)$ , with  $a; b$  real numbers, to the system of equations

$$\begin{aligned} y + jx^j &= 3 \\ jx^j y + x^3 &= 0 \end{aligned}$$

- (A) 1 (B) 2 (C) 3 (D) 5 (E) None of these

7. A contest has ten entries. How many ways are there to choose first, second, and third place, along with two unordered honorable mentions?

- (A) 252 (B) 2520 (C) 15,120 (D) 30,240 (E) None of these

8. Find the absolute value of the sum of all solutions to the equation  $(1 - 2x)(x + 6) = 18$ .

- (A) 3.5 (B) 5.5 (C) 12 (D) 13.5 (E) None of these

9. Which of the following functions is one-to-one on its domain?

- (A)  $f(x) = x^3 - x$  (B)  $f(x) = x^2 + 2$  (C)  $f(x) = e^{x^2}$  (D)  $f(x) = \frac{1}{x+4}$  (E)  $f(x) = x - \frac{1}{x}$

10. How many integers are excluded from the solution set of the inequality  $\frac{3x - 2}{x} > 1$ ?

- (A) Zero (B) One (C) Two (D) Infinitely Many (E) None of these

11. For how many real values of  $x$  is the equation  $(x + 2)^3 = x^3 + 8$  true?

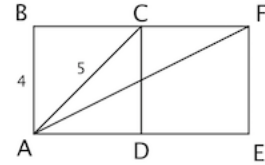
- (A) Zero (B) One (C) Two (D) Infinitely Many (E) None of these

12. The rational expression  $\frac{5x^2 - 2x + 3}{x^3 + 2x^2 + 5x + 10}$  is equivalent to the sum  $\frac{A}{x+2} + \frac{Bx+C}{x^2+5}$ . Find the product  $ABC$ .

(A)  (B) 495 (C) 0 (D) 30 (E) None of these

13. In the figure shown,  $ABCD$  and  $DCFE$  are rectangles, with  $AB = 4$ ,  $AC = 5$ , and  $BC = CF$ . What is the perimeter of  $\triangle ACF$ ?

(A)  (B)  $8 + 5\sqrt{2}$  (C)  $9 + 4\sqrt{5}$   
 (D)  $10 + 5\sqrt{2}$  (E) None of these



14. Find the maximum value of the function  $f(x) = \frac{10}{4x^2 + 12x + 13}$ .

(A) 2 (B)  $\frac{1}{4}$  (C)  $\frac{10}{13}$  (D)  (E) None of these

15. The value of  $\log 3.76$  to four decimal places is 0.5752. Find the value of  $\log 37.6$ .

(A)  (B) 3.627 (C) 5.752 (D) 10.5752 (E) None of these

16. Suppose for all positive integers  $n$ , we have  $f(4 + n^2) = an + 2$  and  $f(9 - n^2) = 3n - b$  for some numbers  $a$  and  $b$ . Then the value of  $f(13)$  is

(A)  (B) 3 (C)

21.

31. Suppose  $f(x)$  is a degree six polynomial with positive leading coefficient, such that  $f(-2) = -1$ . What is the minimum number of real roots of the polynomial  $f(x)$ ?

- (A) 0                      (B) 1                      (C)  2                      (D) 4                      (E) None of these

32. Find the area of the region enclosed by the graphs of the functions  $f(x) = |x|$  and  $g(x) = 4$ .

- (A) 4                      (B) 8                      (C)  16                      (D) 32                      (E) None of these

33. Find the largest integer value of  $n$ , with  $0 < n < 100$ , so that  $(1 + i\sqrt{3})^n$  is a real number.

- (A) 100                      (B)  99                      (C) 98                      (D) 97                      (E) None of these

34.

41. In the figure shown, points  $A$ ,  $B$ ,  $C$ ,  $D$  and  $E$  all lie on the circle, point  $O$  is the center of the circle, and both  $\overline{AD}$  and  $\overline{CE}$  go through point  $O$ . Angle  $\angle BEC$  has measure  $28^\circ$ , and  $\angle ADB$

48. Find the value of the continued fraction

$$\frac{1}{2 + \frac{1}{3 + \frac{1}{2 + \frac{1}{3 + \frac{1}{2 + \dots}}}}}$$

- (A)  $\frac{\sqrt{15} - 3}{2}$  (B)  $\frac{3}{2}$  (C)  $\frac{1 + \sqrt{29}}{14}$  (D)  $\frac{1 + \sqrt{29}}{2}$  (E) None of these

49. Find the equation of the tangent line to the circle  $(x - 1)^2 + (y + 5)^2 = 25$  at the point  $(4; -1)$ .

- (A)  $y = \frac{3}{4}x - 5$  (B)  $y = \frac{3}{4}x + 2$  (C)  $y = \frac{5}{6}x - 5$  (D)  $y = \frac{5}{6}x - \frac{13}{3}$  (E) None of these

50. A right square pyramid with a base area of 16 and a height of 6 is cut halfway up parallel to the base.